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Propagation of Airy-Gaussian beams in a chiral medium

Fu Deng¹, Weihao Yu², Jiayao Huang², Ruihuang Zhao², Jiong Lin², and Dongmei Deng^{1,2,a}

¹ Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials and Devices, South China Normal University, Guangzhou 510631, P.R. China

² CAS Key Laboratory of Geospace Environment, University of Science & Technology of China, Chinese Academy of Sciences, Hefei 230026, P.R. China

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Abstract. We have expressed and investigated the propagation of Airy-Gaussian beams (AiGBs) in a chiral medium analytically. It is shown that AiGBs split into two components, i.e., the left circularly polarized (LCP) beams and the right circularly polarized (RCP) beams, which have a different propagation trajectory and are affected by the chiral parameter γ and the distribution factor χ_0 . It is found that the LCP beams accelerate faster than the RCP beams during propagation, and are influenced by the chiral parameter. With an increase in the chiral parameter, the acceleration of the LCP beams increases, but that of the RCP beams decreases. So, it is significant that we can control the self-acceleration of AiGBs by varying the chiral parameter and the distribution factor.

1 Introduction

Since finite energy Airy beams were first generated by Siviloglou and Chriatodoulides from experiments [1], they have been the hot spot of research [2–7]. In recent years, the Airy-Gaussian beams (AiGBs) which describe in a more realistic way the propagation of the Airy beams, have been widely discussed [8–12]. Because the AiGBs can be regarded as Airy beams with finite power getting through the Gaussian aperture, they carry finite energy and retain their non-diffracting propagation properties within a finite distance of propagation [8]. The propagation of AiGBs through an optical ABCD system [8], in a kerr medium [9], in a strongly nonlocal medium [10], in a quadratic medium [11], and in uniaxial crystals [12] has been reported, respectively.

On the other hand, the chiral medium has many different properties from the ordinary optical medium [13–15]. It is well known that when a linearly polarized beam incidents upon a chiral medium, it will be split into two circularly polarized beams, the left circularly polarized (LCP) beams and the right circularly polarized (RCP) beams, with different phase velocity in the chiral medium [13–15]. There are many reports on the propagation of various kinds of beams through a chiral medium [13–18]. Zhuang et al. studied the evolution of Airy beams in a chiral medium, and they found that the self-acceleration of the beams is sensitive to the interference term [14]. However, there is no report on the propagation of the AiGBs in a chiral medium so far. It is necessary to research the propagation of AiGBs in a chiral medium.

In this paper, the analytical propagation expression of AiGBs through an optical ABCD system is obtained first. Then we obtain the propagation expression of AiGBs through a chiral medium. Finally, the influences of the chiral parameter and the distribution factor χ_0 are discussed.

2 Propagation of Airy-Gaussian beams in a chiral medium

In the coordinate system, the electric field distribution of Airy Gaussian beams at the initial plane can be written as references [2,8,10]

$$E(x_0, y_0, 0) = A_0 \operatorname{Ai}\left(\frac{x_0}{w_1}\right) \operatorname{Ai}\left(\frac{y_0}{w_2}\right) \exp\left(\frac{ax_0}{w_1} + \frac{ay_0}{w_2}\right) \\ \times \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \qquad (1)$$

where a is the decay parameter, A_0 is the constant amplitude of the complex amplitude, w_1 and w_2 denote the arbitrary transverse scales in the x and y directions respectively, and $w_1 = w_2 = \chi_0 w_0$, χ_0 is the distribution factor which determines the beams to be Airy beams when it is small, or Gaussian beams when it is large. Ai(·) denotes the Airy function.

^a e-mail: dmdeng@263.net

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With the Huygens diffraction integral, the AiGBs propagate through an optical ABCD system under the paraxial approximation, which can be determined as reference [19]

$$E(x, y, z) = \frac{ik}{2\pi B} \int \int_{-\infty}^{\infty} E_0(x_0, y_0, 0)$$

$$\times \exp\left\{-\frac{ik}{2B} \left[A\left(x_0^2 + y_0^2\right) - 2\left(x_0 x + y_0 y\right)\right] + D\left(x^2 + y^2\right)\right\} dx_0 dy_0, \qquad (2)$$

where A, B and D are the composition of the transfer matrix and the wave number $k = 2\pi/\lambda$ (λ is the wavelength of beams in the free space). Substituting equation (1) into equation (2), we can obtain the expression of AiGBs propagating through an optical ABCD system as:

$$E(x, y, z) = \frac{iA_0k}{2BM} \exp\left(P(x, y, z)\right) Ai(Q(x)) Ai(Q(y)),$$
(3)

where

$$P(x, y, z) = -\frac{ikD}{2B} (x^2 + y^2) - \frac{k^2}{4B^2M} (x^2 + y^2) + \frac{ik}{8BM^2} \\ \times \left(\frac{x}{w_1^3} + \frac{y}{w_2^3}\right) + \frac{iak}{2BM} \left(\frac{x}{w_1} + \frac{y}{w_2}\right) + \frac{1}{96M^3} \\ \times \left(\frac{1}{w_1^6} + \frac{1}{w_2^6}\right) + \frac{a}{8M^2} \left(\frac{1}{w_1^4} + \frac{1}{w_2^4}\right) \\ + \frac{a^2}{4M} \left(\frac{1}{w_1^2} + \frac{1}{w_2^2}\right),$$
(4)

$$Q(x) = \frac{ikx}{2BMw_1} + \frac{a}{2Mw_1^2} + \frac{1}{16M^2w_1^4},$$
 (5)

$$Q(y) = \frac{iky}{2BMw_2} + \frac{a}{2Mw_2^2} + \frac{1}{16M^2w_2^4},$$
 (6)

with $M = \frac{1}{w_0^2} + \frac{ikA}{2B}$.

Following, the chiral medium as an ABCD system can be written as references [13-15]

$$\begin{pmatrix} A^{(L)} & B^{(L)} \\ C^{(L)} & D^{(L)} \end{pmatrix} = \begin{pmatrix} 1 & z/n^{(L)} \\ 0 & 1 \\ \\ 1 & z/n^{(R)} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A^{(R)} & B^{(R)} \\ C^{(R)} & D^{(R)} \end{pmatrix}$$
(7)

where $n^{(L)} = n/(1+nk\gamma)$ and $n^{(R)} = n/(1-nk\gamma)$ indicate the refractive indexes of LCP beams and RCP beams, respectively. γ is the parameter of the chiral medium and nis the original refractive index. Substituting equation (7) into equation (3), we can get the analytical expression of the two components of AiGBs through the chiral medium as $E^{(L)}(x, y, z)$ and $E^{(R)}(x, y, z)$. The total electric field is references [13–15]

$$E = E^{(L)}(x, y, z) + E^{(R)}(x, y, z).$$
 (8)

So the total intensity of AiGBs in the chiral medium can be written as references [13-15]

$$I = |E^{(L)}(x, y, z)|^2 + |E^{(R)}(x, y, z)|^2 + I_{int}, \qquad (9)$$

with

$$I_{int} = E^{(L)}(x, y, z) E^{(R)*}(x, y, z) + E^{(R)}(x, y, z) \times E^{(L)*}(x, y, z),$$
(10)

where I_{int} is the interference term of AiGBs in the chiral medium and * represents the complex conjugate of $E^{(L)}(x, y, z)$ and $E^{(R)}(x, y, z)$.

We will also investigate the dynamics of the center of gravity of the AiGBs in a chiral medium, coming from the intensity distribution [8,10,11],

$$X_c = \frac{\iint_{-\infty}^{\infty} x I dx dy}{\iint_{-\infty}^{\infty} I dx dy}.$$
 (11)

3 Numerical calculations and analysis

Having found the analytical results in equations (8)–(10) and the center of gravity in equation (11), we will discuss that the simulations of intensity are calculations based on the analytical results and the center of gravity, based on simulations described in the following. Based on studies of AiGBs [8–12] and on the propagation of beams in a chiral medium [13–15], here we concentrate our attention on the influence of the chiral parameter γ and the distribution factor χ_0 of the AiGBs.

In the following simulations, some parameters are chosen as $A_0 = 1$, $\lambda = 633$ nm, $w_0 = 0.1$ mm, a = 0.05, n = 3and Zr is the Rayleigh range, which is given $Zr = kw_1^2/2$. From equations (3)–(10), we can see that $E^{(L)}(x, y, z)$, $E^{(R)}(x, y, z)$ and the interference term are sensitive to the propagating distance z, the chiral parameter γ of the medium, and the distribution factor χ_0 . The interference term with different chiral parameters γ and the distribution factor χ_0 is shown in Figure 1. The peak intensity of the interference term decreases with the propagating distance, the chiral parameter and the distribution factor increasing.

The propagation of AiGBs through a chiral medium with $\gamma = 0.16/k$, $\chi_0 = 0.1$ is shown in Figure 2. We can easily see that the LCP beams and RCP beams have a different propagation trajectory from Figures 2a1, 2a2 and 2b1-2b4, because the LCP beams not only selfaccelerate but also diffract faster than the RCP beams during propagation in the chiral medium. We can also find that the total intensity is distorted due to the difference between the two beams and the interference term. In order to find the influence of the chiral parameter, the propagation of AiGBs through a chiral medium with $\gamma = 0.28/k$ is shown in Figure 3. We can find that when the chiral parameter increases, the self-acceleration of the two circularly polarized beams changes oppositely. The self-acceleration of the LCP beams increases but that of



Fig. 1. The interference term with different chiral parameter and distribution factor χ_0 , (a1)–(a3) denote $\gamma = 0.16/k$ (x = y), (b1)–(b3) denote $\gamma = 0.28/k$, (a1) and (b1) denote $\chi_0 = 0.05$, (a2) and (b2) denote $\chi_0 = 0.1$, (a3) and (b3) denote $\chi_0 = 0.3$.

l l(a1) l	x (mm)	l l (a2) l		I (a3) 1 —	
1 I 0 10 :	20 30 z/Zr	Q 10 20 30		0 10 20 30 -1 -	
(b1)	(b2)	(b3)	(b4)	(b5)	(b6)
y (mm)	1 <u>mm</u>				18

Fig. 2. Numerical demonstrations of the AiGBs propagating through the chiral medium with $\gamma = 0.16/k$, $\chi_0 = 0.1$. (a1)–(a3) denote the LCP, RCP and total intensity respectively (x = y). (b1)–(b6) are the transverse intensity patterns taken by the dotted lines in (a1)–(a3), where (b1), (b3) and (b5) are for z = 2Zr, and (b2), (b4) and (b6) are for z = 10Zr, respectively.

(a1)	x (mm)	(a2)		(a3)	1 -
0 10 2	z/Zr	0 10 20) 30	0 10 2	0
(b1) y (mm) t x (mm)	(b2) 1mm	(b3)	(b4)	(b5)	(b6)

Fig. 3. Numerical demonstrations of the AiGBs propagating through the chiral medium with $\gamma = 0.28/k$. All parameters are the same as in Figure 2.



Fig. 4. The intensity distribution of the AiGBs propagating in the chiral medium ($\gamma = 0.16/k$) at different observation sections with different χ_0 . (a1)–(a3) $\chi_0 = 0.05$, (b1)–(b3) $\chi_0 = 0.1$ and (c1)–(c3) $\chi_0 = 0.3$. (a1)–(c1) represent the LCP intensity, (a2)–(c2) represent the RCP intensity and (a3)–(c3) represent the total intensity.



Fig. 5. The intensity distribution of the AiGBs ($\chi_0 = 0.1$) propagating in the chiral medium with the effects of different γ . (a1)-(a4) are at z = 2Zr, (b1)-(b4) are at z = 10Zr. The first column denotes the LCP intensity, the second column denotes the RCP intensity, the third column denotes the interference term intensity and the fourth column denotes the total intensity.

the RCP beams decreases. It is interesting to find that the self-acceleration of the total intensity also decreases with an increase in the chiral parameter. So we can control the self-acceleration of the AiGBs in the chiral medium by controlling the chiral parameter γ of the chiral medium.

In order to find the influence of the distribution factor χ_0 , the intensity distribution of the AiGBs propagating in the chiral medium at different observation sections with different χ_0 is shown in Figure 4. By comparing the propagation Airy beams in a chiral medium (see Ref. [14]), we can find that the propagation Airy beams in a chiral medium is a special case of the propagation AiGBs in a chiral medium. We can also find that the beams diffract more quickly with increases in the distribution factor χ_0 , meaning that when the AiGBs tend to be the Gaussian beams, they diffract faster through the chiral medium.

The intensity distribution of the AiGBs ($\chi_0 = 0.1$) propagating in the chiral medium with the effects of different γ in the near-zone and far-zone is shown in Figure 5. In the near-zone, the LCP beams, the RCP beams, the interference term and the total field are hardly influenced by the different chiral parameters. But in the far-zone, the LCP beams will change with different γ . The interference term and the total intensity become more complicated than that in the near-zone, and they will be also influenced by the different chiral parameters. The intensity distribution of the AiGBs ($\gamma = 0.16/k$) propagating in the chiral medium with the effects of different χ_0 in the near-zone and far-zone is shown in Figure 6. With the large distribution factor χ_0 , the wave form of the AiGBs in both the near-zone and the far-zone becomes steadier during propagation in the chiral medium. Finally,



Fig. 6. The intensity distribution of the AiGBs ($\gamma = 0.16/k$) propagating in the chiral medium with the effects of different χ_0 . (a1)-(a4) are at z = 2Zr, (b1)-(b4) are at z = 10Zr. The first column denotes the LCP intensity, the second column denotes the RCP intensity, the third column denotes the interference term intensity and the fourth column denotes the total intensity.



Fig. 7. The centre of gravity of AiGBS in the chiral ($\gamma = 0.16/k$) in the x-direction with different χ_0 .

the position of the centroid of the total intensity of AiGBs in the chiral medium ($\gamma = 0.16/k$) with different χ_0 is shown in Figure 7. We can find that the center of gravity is influenced by the distribution factor, and is more steadier during propagation in the chiral medium with the large χ_0 .

4 Conclusions

In conclusion, we have expressed and investigated the propagation of AiGBs in a chiral medium. It is shown that AiGBs split into two components, i.e., the LCP beams and the RCP beams, which have a different propagation trajectory. Both are affected by the chiral parameter γ and the distribution factor χ_0 . It is found that the LCP beams accelerate faster than the RCP beams during propagation, and that they are influenced by the chiral parameter.

With increases in the chiral parameter, the acceleration of the LCP beams increases, but that of the RCP beams decreases. It is interesting that when increasing the distribution factor χ_0 , the beams will diffract quickly. So it is significant that we can control the self-acceleration of AiGBs by controlling the chiral parameter γ and the distribution factor χ_0 . We believe that the properties of AiGBs in a chiral medium may have various applications such as in optical communications.

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