

Propagation of Airy Gaussian vortex beams in uniaxial crystals*

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The propagation dynamics of the Airy Gaussian vortex beams in uniaxial crystals orthogonal to the optical axis has been investigated analytically and numerically. The propagation expression of the beams has been obtained. The propagation features of the Airy Gaussian vortex beams are shown with changes of the distribution factor and the ratio of the extraordinary refractive index to the ordinary refractive index. The correlations between the ratio and the maximum intensity value during the propagation, and its appearing distance have been investigated.

Keywords: Airy Gaussian vortex beams, uniaxial crystals, anisotropic effect

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1. Introduction

In 1979, Berry and Balazs introduced the nonspreading Airy wave packets by solving Schrödinger equation.^[1] The packets bring many researchers' interests due to their unique properties of nonspreading and constant acceleration in free space. In 2007, on the basis of previous studies, Siviloglou *et al.* obtained finite energy Airy beams by adding a decay factor. They investigated and observed those beams in both one- and two-dimensional configurations theoretically^[2] and experimentally,^[3] finding that the finite energy Airy beams also preserve quasi-diffraction-free and free acceleration properties. Next year, self-healing properties were investigated by John Broky *et al.*^[4] Then Airy beams were widely investigated in many kinds of materials such as free space,^[5–7] right-handed material to left-handed material,^[8] bulk nonlinear media,^[9–15] and a quadratic-index medium.^[16] Nowadays, researches on Airy beams are involved in various fields of military,^[17–19] micro–nano technology,^[20–23] atmospheric sciences,^[24] and so on.

Furthermore, it is an interesting subject to describe the light propagation in the anisotropic media in both theoretical and applied optics.^[25,26] In reality, crystals play an important part in the design of optical devices, e.g., polarizers and compensators, because of their ability to affect the polarization state of light.^[27] Through uniaxial crystals, the propagation of Airy beams,^[28] Airy vortex beams,^[29] and Airy Gaussian beams^[30] has been investigated.

Airy Gaussian vortex beams (AiGVBs) are obtained from Airy beams multiplied Gaussian factor and vortex factor. It is intriguing for AiGVBs that these beams not only have the unique features of Airy Gaussian beams:^[31,32] free acceleration and self-healing, but also have the properties of vortex beams:^[33,34] intensity singularities and phase singularities. However, to the best of our knowledge, AiGVBs only have been investigated in the media of right-hand materials and left-hand materials.^[34] Therefore, in the rest of the paper, the propagation of AiGVBs in uniaxial crystals is to be investigated.

2. Propagation of Airy Gaussian vortex beams in uniaxial crystals

In the spatial coordinate system, the z axis is taken to be the propagation axis and the x axis is taken to be the optical axis of the uniaxial crystal. The observation plane is taken to be z and the input plane is $z = 0$. The relative dielectric tensor ϵ of the uniaxial crystal is set as

$$\epsilon = \begin{bmatrix} n_e^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_o^2 \end{bmatrix}, \quad (1)$$

where n_e and n_o are the extraordinary and the ordinary refractive indices of the uniaxial crystal. The electric field distribution of the AiGVBs in the input plane $z = 0$ reads

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$$\begin{bmatrix} E_x(x_0, y_0, 0) \\ E_y(x_0, y_0, 0) \end{bmatrix} = \begin{bmatrix} A_0 \text{Ai}\left(\frac{x_0}{\chi_0 w}\right) \exp\left(\frac{a_x x_0}{\chi_0 w}\right) \text{Ai}\left(\frac{y_0}{\chi_0 w}\right) \exp\left(\frac{a_y y_0}{\chi_0 w}\right) \exp\left(-\frac{x_0^2 + y_0^2}{w^2}\right) \\ \times \left(\frac{x_0 - x_1}{\chi_0 w} + i \frac{y_0 - y_1}{\chi_0 w}\right)^m \left(\frac{x_0 - x_2}{\chi_0 w} - i \frac{y_0 - y_2}{\chi_0 w}\right)^n \\ 0 \end{bmatrix}, \quad (2)$$

where $E_x(x_0, y_0, 0)$ and $E_y(x_0, y_0, 0)$, respectively, stand for the initial electric field distribution in the x and y directions; A_0 is the amplitude of the beams; $\text{Ai}(\cdot)$ is the Airy function; χ_0 is a distribution factor which can be non-zero real number (for simplicity, we only value it positive real number in this paper); a_x and a_y , respectively, stand for decay factors in the x and y directions which make the energy of the beams be finite; w stands for the beam width of Gaussian beams;

$\left(\frac{x_0 - x_1}{\chi_0 w} + i \frac{y_0 - y_1}{\chi_0 w}\right)^m$ is the positive vortex factor, while $\left(\frac{x_0 - x_2}{\chi_0 w} - i \frac{y_0 - y_2}{\chi_0 w}\right)^n$ is the negative one, m and n are orders of their factor, respectively, x_1, y_1 and x_2, y_2 , respectively, stand for positions from the center of the positive and negative vortex factors. However, for simplicity, in this paper, we only discuss a situation with $m = 1, n = 0$, and $x_1 = y_1 = x_2 = y_2 = 0$. Hence, the initial electric field distribution of AiGVs in this paper reads

$$\begin{bmatrix} E_x(x_0, y_0, 0) \\ E_y(x_0, y_0, 0) \end{bmatrix} = \begin{bmatrix} A_0 \text{Ai}\left(\frac{x_0}{\chi_0 w}\right) \exp\left(\frac{a_x x_0}{\chi_0 w}\right) \text{Ai}\left(\frac{y_0}{\chi_0 w}\right) \exp\left(\frac{a_y y_0}{\chi_0 w}\right) \\ \times \exp\left(-\frac{x_0^2 + y_0^2}{w^2}\right) \left(\frac{x_0}{\chi_0 w} + i \frac{y_0}{\chi_0 w}\right) \\ 0 \end{bmatrix}. \quad (3)$$

Under the paraxial approximation, the propagation formulas of the AiGVs orthogonal to the axis can be obtained by^[27,28,35]

$$E_x(x, y, z) = \frac{ikn_0}{2\pi z} \exp(-ikn_0 z) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x_0, y_0, 0) \exp\left\{-\frac{ik}{2zn_0} [n_0^2(x-x_0)^2 + n_0^2(y-y_0)^2]\right\} dx_0 dy_0, \quad (4)$$

$$E_y(x, y, z) = \frac{ikn_0}{2\pi z} \exp(-ikn_0 z) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y(x_0, y_0, 0) \exp\left\{-\frac{ikn_0}{2z} [(x-x_0)^2 + (y-y_0)^2]\right\} dx_0 dy_0, \quad (5)$$

where $k = 2\pi/\lambda$ is the wave number and λ is the optical wavelength. Substituting Eqs. (1) and (3) into Eqs. (4) and (5), and using Airy integral formulas

$$\int \text{Ai}\left(\frac{x}{a}\right) \exp(bx^2 + cx) dx = \sqrt{-\frac{\pi}{b}} \exp\left(-\frac{c^2}{4b} + \frac{c}{8a^3 b^2} - \frac{1}{96a^6 b^3}\right) \text{Ai}\left(\frac{1}{16a^4 b^2} - \frac{c}{2ab}\right), \quad (6)$$

$$\begin{aligned} \int x \text{Ai}\left(\frac{x}{a}\right) \exp(bx^2 + cx) dx &= \sqrt{-\frac{\pi}{b}} \exp\left(-\frac{c^2}{4b} + \frac{c}{8a^3 b^2} - \frac{1}{96a^6 b^3}\right) \\ &\times \left[\left(-\frac{c}{2b} + \frac{1}{8a^3 b^2}\right) \text{Ai}\left(\frac{1}{16a^4 b^2} - \frac{c}{2ab}\right) - \frac{1}{2ab} \text{Ai}'\left(\frac{1}{16a^4 b^2} - \frac{c}{2ab}\right)\right], \end{aligned} \quad (7)$$

the analytical complex field of AiGVs after propagating a distance z in uniaxial crystals orthogonal to the optical axis can be obtained as

$$E_x(x, y, z) = G_0(G_1 G_2 + G_3 G_4), \quad (8)$$

$$E_y(x, y, z) = 0, \quad (9)$$

where

$$G_0 = \frac{ikn_0}{2\pi z} A_0 \exp(-ikn_0 z), \quad (10)$$

$$G_1 = \exp\left(\frac{-ikn_0^2 x^2}{2zn_0}\right) \frac{1}{\chi_0 w} \sqrt{-\frac{\pi}{b_1}}$$

$$\begin{aligned} &\times \exp\left(-\frac{c_1^2}{4b_1} + \frac{c_1}{8a^3 b_1^2} - \frac{1}{96a^6 b_1^3}\right) \\ &\times \left[\left(-\frac{c_1}{2b_1} + \frac{1}{8a^3 b_1^2}\right) \text{Ai}\left(\frac{1}{16a^4 b_1^2} - \frac{c_1}{2ab_1}\right) - \frac{1}{2ab_1} \text{Ai}'\left(\frac{1}{16a^4 b_1^2} - \frac{c_1}{2ab_1}\right)\right], \end{aligned} \quad (11)$$

$$\begin{aligned} G_2 &= \exp\left(\frac{-ikn_0 y^2}{2z}\right) \sqrt{-\frac{\pi}{b_2}} \\ &\times \exp\left(-\frac{c_2^2}{4b_2} + \frac{c_2}{8a^3 b_2^2} - \frac{1}{96a^6 b_2^3}\right) \\ &\times \text{Ai}\left(\frac{1}{16a^4 b_2^2} - \frac{c_2}{2ab_2}\right), \end{aligned} \quad (12)$$

$$G_3 = \exp\left(\frac{-ikn_o^2 x^2}{2zn_e}\right) \sqrt{\frac{\pi}{b_1}} \times \exp\left(-\frac{c_1^2}{4b_1} + \frac{c_1}{8a^3 b_1^2} - \frac{1}{96a^6 b_1^3}\right) \times \text{Ai}\left(\frac{1}{16a^4 b_1^2} - \frac{c_1}{2ab_1}\right), \quad (13)$$

$$G_4 = \exp\left(\frac{-ikn_e y^2}{2z}\right) \frac{i}{\chi_0 w} \sqrt{\frac{\pi}{b_2}} \times \exp\left(-\frac{c_2^2}{4b_2} + \frac{c_2}{8a^3 b_2^2} - \frac{1}{96a^6 b_2^3}\right) \times \left[\left(-\frac{c_2}{2b_2} + \frac{1}{8a^3 b_2^2}\right) \text{Ai}\left(\frac{1}{16a^4 b_2^2} - \frac{c_2}{2ab_2}\right) - \frac{1}{2ab_2} \text{Ai}'\left(\frac{1}{16a^4 b_2^2} - \frac{c_2}{2ab_2}\right) \right]. \quad (14)$$

As for expressions (11)–(14), where

$$a = \chi_0 w, \quad (15a)$$

$$b_1 = -\left(\frac{1}{w^2} + \frac{ikn_o^2}{2zn_e}\right), \quad (15b)$$

$$c_1 = \left(\frac{a_x}{\chi_0 w} + \frac{ikn_o^2 x}{zn_e}\right), \quad (15c)$$

$$b_2 = -\left(\frac{1}{w^2} + \frac{ikn_e}{2z}\right), \quad (15d)$$

$$c_2 = \left(\frac{a_y}{\chi_0 w} + \frac{ikn_e y}{z}\right). \quad (15e)$$

3. Numerical calculations and analyses

Here we investigate how the changes of χ_0 and n_e/n_o affect the propagation of AiGVs in uniaxial crystals. The beam parameters are chosen as follows: $\lambda = 530$ nm, $a_x = a_y = 0.1$, the normalized coefficient $X_0 = \lambda w = 10^{-4}$ m, the Rayleigh distance $Z_0 = kX_0^2/2 \approx 6$ cm, and $n_o = 2.616$. Hereafter, these parameters will not change.

First, we will consider the case of different χ_0 . We set $n_o = 3.1392$ and intensity $I(x, y, z) = |E_x(x, y, z)|^2$. At each observation plane, we normalize the values of intensity with

$$\frac{I(x, y, z) - I(x, y, z)_{\min}}{I(x, y, z)_{\max} - I(x, y, z)_{\min}}, \quad (16)$$

where $I(x, y, z)$ means the value of the intensity at the observation plane z , and $I(x, y, z)_{\max}$ or $I(x, y, z)_{\min}$ means the maximum or the minimum value of the intensity at that plane.

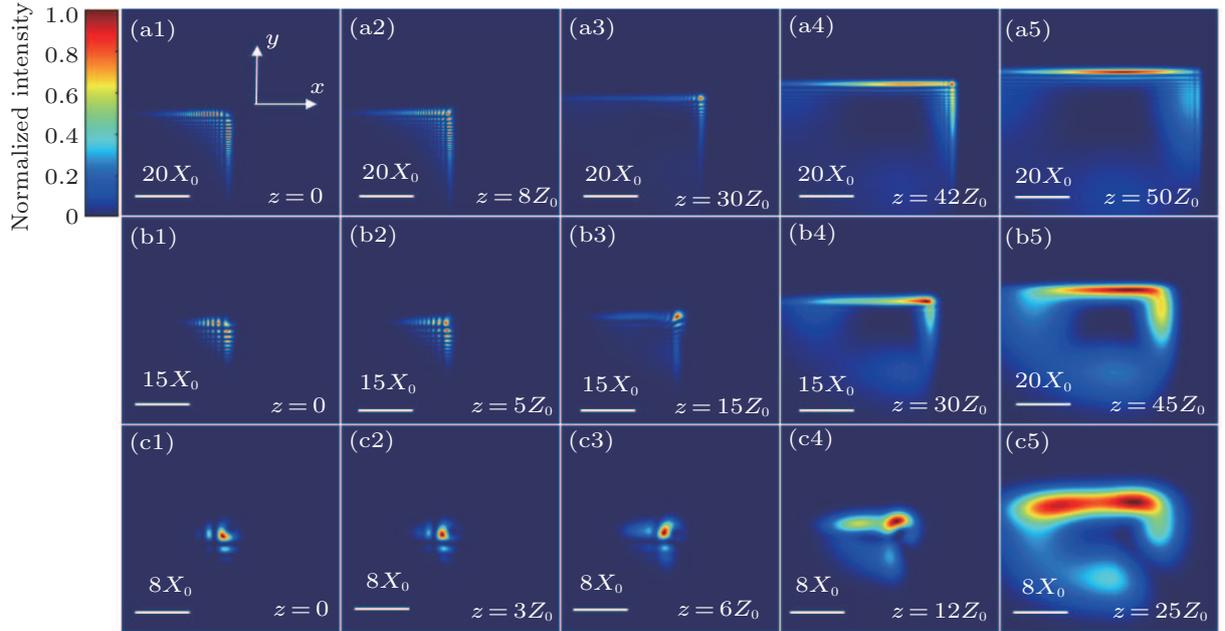


Fig. 1. (color online) Normalized intensity distribution of AiGVs propagating in the uniaxial crystals at several observation planes. (a1)–(a5) $\chi_0 = 0.01$, (b1)–(b5) $\chi_0 = 0.1$, and (c1)–(c5) $\chi_0 = 0.3$.

From Figs. 1 and 2, we can find that if χ_0 takes smaller number, the distributions of the intensity and the phase approach the distributions of Airy vortex beams, like Figs. 1(a1) and 2(a1), while if χ_0 takes larger number, the distributions approach those of Gaussian vortex beams, like Figs. 1(a3) and 2(a3). The smaller χ_0 is, the more largely the Airy factor affects, and on the contrary, the more largely the Gaussian factor affects. The Gaussian factor can strengthen main lobes and weaken side lobes, but the vortex factor can weaken main lobes, like Figs. 1(a1), 1(b1), and 1(c1). In the propagation

process, figures 1(a2), 1(b2), and 1(c2) show that AiGVs heal firstly and each main lobe is rebuilt when $z = 8Z_0$, $z = 5Z_0$, and $z = 3Z_0$, demonstrating that the healing distance decreases as χ_0 increases. After healing, main lobes further strengthen and the energy of side lobes converges into main lobes until most energy is concentrated on the main lobes, like Figs. 1(a3), 1(b3), and 1(c3). Then the energy flows along the x and y directions, but the energy flows along the x direction more largely due to $n_e > n_o$. In further propagation, the energy mostly distributes along the x direction.

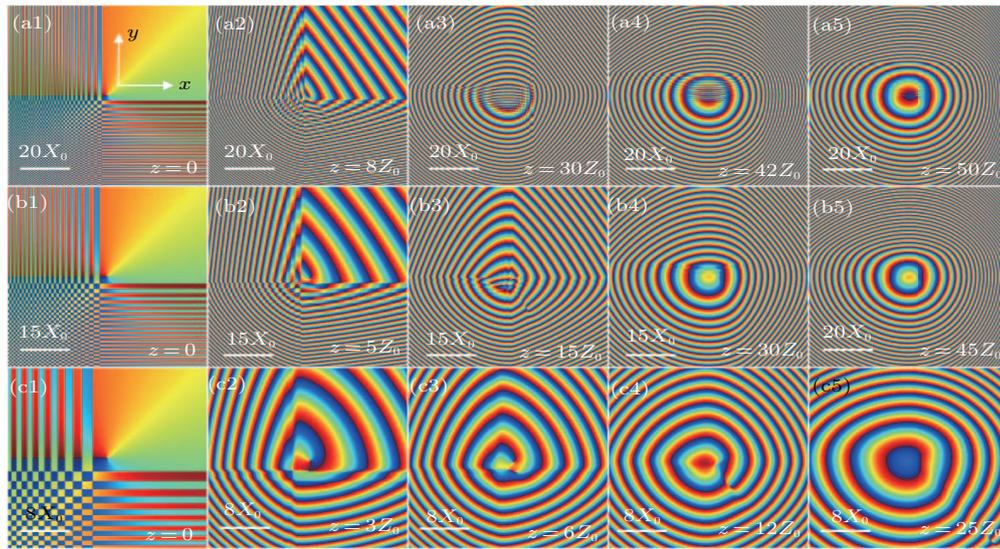


Fig. 2. (color online) Phase distribution of AiGVBs propagating in the uniaxial crystals at several observation planes. (a1)–(a5) $\chi_0 = 0.01$, (b1)–(b5) $\chi_0 = 0.1$, (c1)–(c5) $\chi_0 = 0.3$.

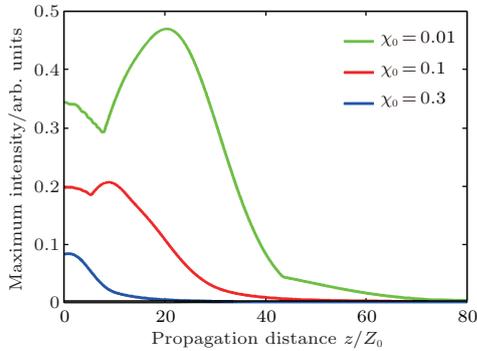


Fig. 3. (color online) Maximum intensity of each observation plane (different z) of the AiGVBs with different χ_0 .

Then, we investigate the maximum intensity of each ob-

servation plane (different z) of the AiGVBs with different χ_0 (see Fig. 3). It shows that if χ_0 is smaller, the beams approach Airy vortex beams, so the maximum intensity firstly increases as the distance increases, corresponding to the healing process. If χ_0 is larger, the effect of the Gaussian factor enhances, causing the beams to diffract rapidly, so the maximum intensity decreases rapidly.

Next, we will investigate how the change of n_e/n_o affects the propagation of AiGVBs in uniaxial crystals. Here, we set $\chi_0 = 0.01$ and $n_o = 2.616$. The normalized intensity and the phase distributions with different values of n_e/n_o are shown in Figs. 4 and 5.

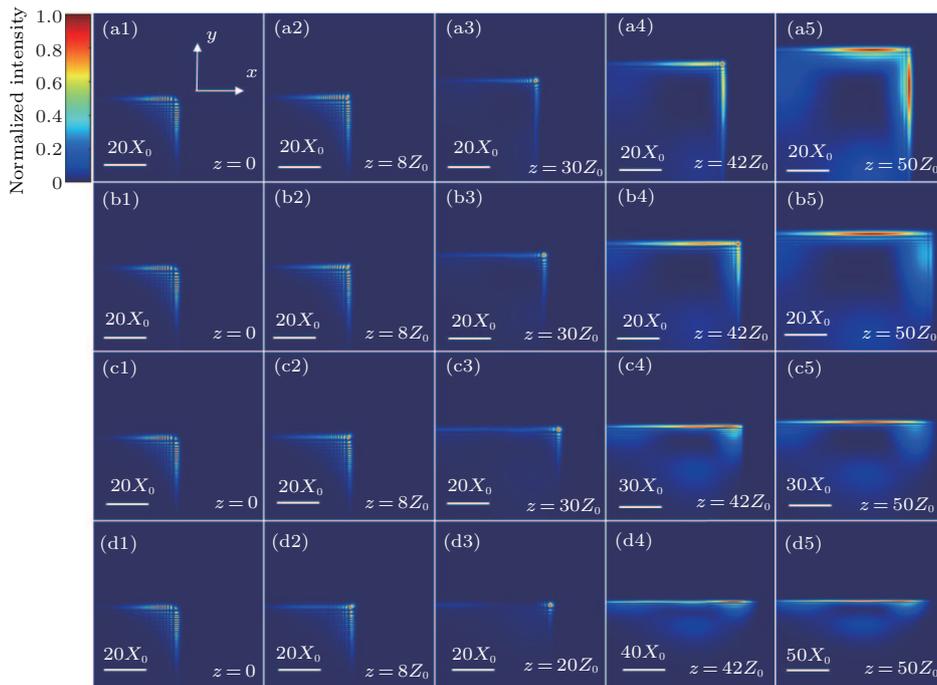


Fig. 4. (color online) Normalized intensity distribution of AiGVBs propagating in the uniaxial crystals at several observation planes. (a1)–(a5) $n_e/n_o = 1$, (b1)–(b5) $n_e/n_o = 1.2$, (c1)–(c5) $n_e/n_o = 1.5$, and (d1)–(d4) $n_e/n_o = 2$.

The two figures show that the value of n_e/n_0 has a great impact on the distributions of the intensity and the phase. Figures 4(a1)–4(a4) show that if $n_e = n_0$, the intensity distribution along the x direction equals that in the y direction. As

the value of n_e/n_0 decreases, the energy more obviously distributes along the x direction. As for the phase, figure 5 shows that as the value of n_e/n_0 decreases, the phase looks more like an ellipse spiral shape.

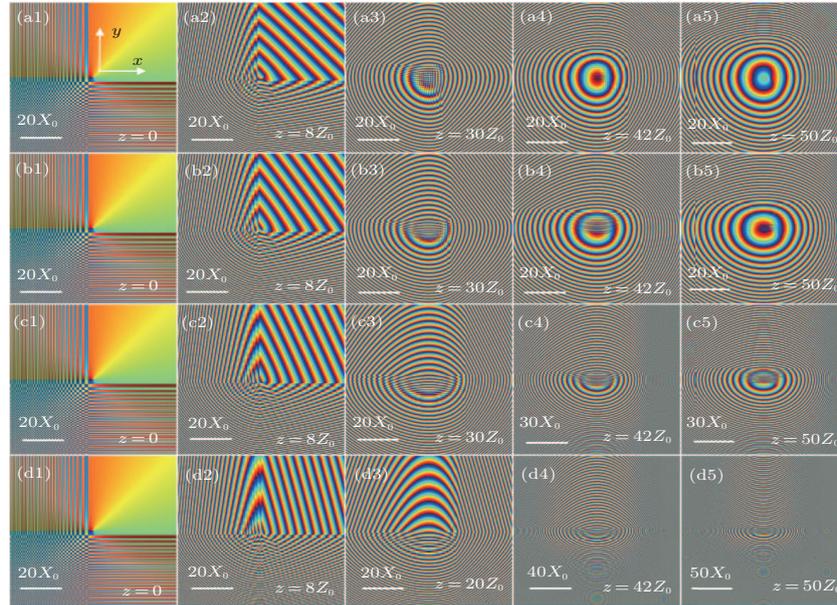


Fig. 5. (color online) Phase distribution of AiGVBs propagating in the uniaxial crystals at several observation planes. (a1)–(a5) $n_e/n_0 = 1$, (b1)–(b5) $n_e/n_0 = 1.2$, (c1)–(c5) $n_e/n_0 = 1.5$, and (d1)–(d4) $n_e/n_0 = 2$.

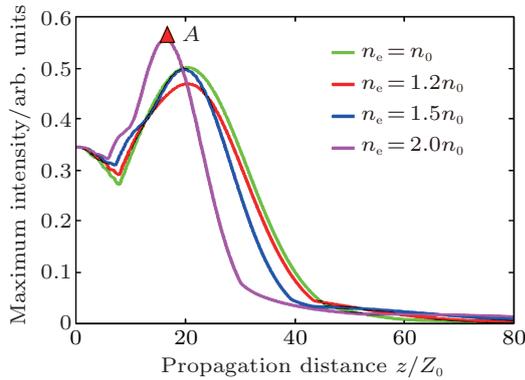


Fig. 6. (color online) Maximum intensity of each observation plane (different z) of the AiGVBs with different values of n_e/n_0 .

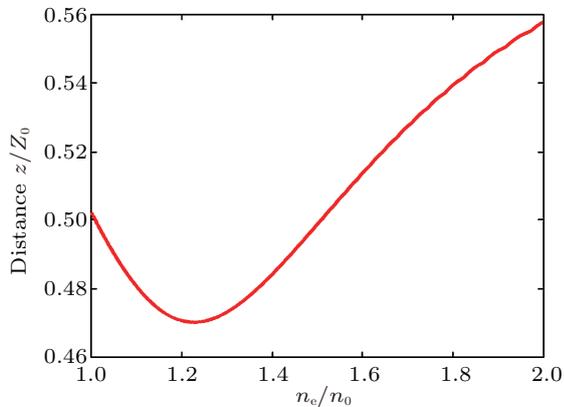


Fig. 7. Maximum intensity value during propagation with different values of n_e/n_0 .

We continue to investigate the maximum intensity values of each observation plane (different z) of the AiGVBs with dif-

ferent values of n_e/n_0 . Some results are shown in Fig. 6. We find that the maximum intensity value during the propagation and its appearing distance z are not monotonic with the ratio of n_e/n_0 . As for what are the maximum intensity value during the propagation and its appearing distance z , for example, in the propagation of the beam with $n_e = 2.0n_0$ in Fig. 6, the values on horizontal and vertical coordinates that the point A corresponds to are the maximum value and its appearing distance z . We further investigate, and the results are showed in Figs. 7 and 8.

Figure 7 shows that as n_e/n_0 increases, the maximum intensity value during the propagation firstly decreases then increases. The minimum intensity appears when $n_e = 1.23n_0$. As discussed above, Airy factor makes the energy of the AiGVBs concentrate to the center while the increase of n_e/n_0 makes the energy more distribute along with the x direction. Although these two effects both concentrate the energy, to some extent, the direction of concentrating the energy of the two effects is different. The increase of n_e/n_0 firstly will weaken the effect of Airy factor of concentrating the energy to the center, so the maximum intensity value during the propagation decreases as n_e/n_0 increases firstly. Then, as n_e/n_0 increases, the effect of n_e/n_0 becomes larger than the effect of Airy factor, so after $n_e/n_0 = 1.23$, the maximum intensity value increases with the increase of n_e/n_0 . The correlation between the appearing distance z of the maximum value and n_e/n_0 is also not monotonic, too. The general trend is that the appearing distance z of the maximum value firstly decreases,

next increases, and then decreases as n_e/n_o increases.

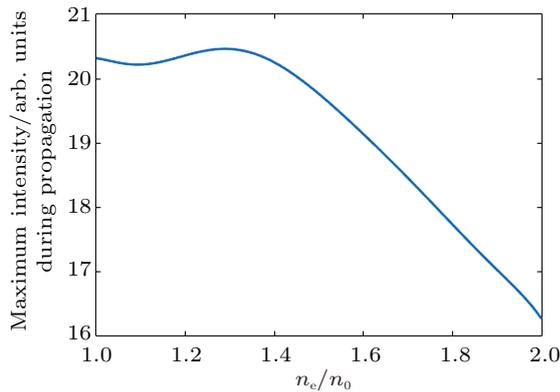


Fig. 8. Appearing distance z of the maximum intensity value during propagation with different values of n_e/n_o .

4. Conclusions

The propagation dynamics of the Airy Gaussian vortex beams in uniaxial crystals orthogonal to the optical axis has been investigated analytically and numerically. The propagation expression of the beams has been obtained. The propagation features of the beams with changes of the distribution factor χ_0 and the ratio of the extraordinary refractive index n_e to the ordinary refractive index n_o are showed. When χ_0 is valued smaller, the distributions of the intensity and the phase approach to the distributions of the Airy vortex beams, and on the contrary, the distributions approach to those of the Gaussian vortex beams. The ratio n_e/n_o affects the distributions of the intensity and the phase, as well as the maximum intensity value of each observation plane, the maximum intensity value during the propagation and its appearing distance. However, the correlations between the ratio and the maximum intensity value during the propagation, between the ratio and the appearing distance are not monotonic.

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